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t: VISCOUS INTERACTION BETWEEN THE SOLAR WIND
AND THE EARTH'S MAGNETOSPHERE.

W. I. Axford Sep. 1963 29p refs

Abstract

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Order of magnitude calculations are used to show that viscous interaction between the solar wind and the earth's magnetosphere can satisfy the energy requirements of a typical magnetic storm. The viscous interaction is considered to be due mainly to turbulence of a compressible nature in the solar wind, and it is shown that this can provide the necessary drag forces, although other mechanisms are not excluded. AUTHOR

1. Introduction

According to the theory advanced by Axford and Hines[1] (hereafter referred to as I), the aurora and associated phenomena are the direct result of a "viscous-like" interaction between the solar wind and the earth's magnetosphere. Several aspects of geomagnetic storms are considered to be simply an enhancement of this interaction, which should occur to some extent even during geomagnetically quiet periods. It is suggested in I that the viscous-like interaction causes a circulation to be set up within the magnetosphere rather similar to that which occurs in a falling raindrop, but being of the interchange type described by Gold[2]. The streamlines of the proposed circulation are sketched in figure (1) for the equatorial section of the magnetosphere; these are also the equipotentials of a corresponding electric field. (For the purposes of this paper, we will ignore the possible asymmetry in the circulation due to high rotational speeds of the surface of the magnetosphere[3].)

There are obvious merits to this suggestion. In the first place, the circulation provides a simple and effective accelerating mechanism which can energize captured solar wind particles from 1 kev up to about 10 kev, corresponding to conservation of magnetic moment between fields of about 50_γ at the surface of the magnetosphere and 500_γ at a geocentric distance of 4 R_e (R_e = earth radius). (Hines[4] has recently given a more detailed account of the acceleration mechanism than that given in I.) Secondly the polar current system D_s[5] and the pattern of alignment of auroral arcs at high latitudes[6] are reproduced by the circulation at ionospheric levels (figure (2)), giving us some understanding of the current systems and their relationship to the motion of auroral irregularities. Thirdly, by combining the circulation with the motion associated with the rotation of the earth, it is possible to predict qualitatively much of the morphology of high latitude disturbance phenomena. Finally, the circulation in combination with irreversible accelerating processes provides an input mechanism for the outer radiation belt.

As pointed out by Gold[7], some viscous interaction between the solar wind and the surface of the earth's magnetosphere must be expected to occur. However, it has not yet been shown that the interaction is sufficiently strong to produce the required effects. Consequently Fejer[8,9] has suggested various alternative mechanisms involving the geomagnetically trapped corpuscular radiation. In the present paper an attempt is made to examine the viscous interaction hypothesis more closely than in I, and by simple order of magnitude calculations it is shown that viscous interaction is an entirely plausible mechanism and gives results which are consistent with observations.

If the surface of the magnetosphere were unstable as has been suggested in the past by Dungey[10], Parker[11] and others, then very effective viscous interaction would result from turbulent mixing of the solar wind and the outer fringes of the magnetosphere. It seems quite possible that the surface of the magnetosphere is in fact relatively stable, as has been argued by Dessler [12,13]; however, the mechanism for momentum transfer advocated here does not involve turbulent mixing and so is not adversely affected by the absence of any instability. Nevertheless the question of the stability of the surface of the magnetosphere should not be regarded as closed. It is unlikely that the surface should suffer from Helmholtz instability on the sunwards side where the streaming velocity of the interplanetary gas is low[14]. However, the instability might occur on the flanks of the magnetosphere where the streaming velocity is large, and it is possible that the "flapping" of the boundary observed by Explorer X[15] is an indication that such is the case. Rayleigh-Taylor instability resulting from normal acceleration of the surface due to changes in the solar wind pressure is also a possibility, although the curvature of the field lines is a stabilizing factor to some extent[16].

Observations of the magnetic field beyond the boundary of the magnetosphere have indicated the presence of a highly turbulent plasma [15,17,18,19]. This has been interpreted by Axford[16] as being due to a shock wave which is set up on the sunwards side of the magnetosphere by the highly supersonic solar wind. The turbulence can be considered as a random assemblage of hydromagnetic waves containing both longitudinal and transverse modes. It is suggested that the longitudinal waves (which are effectively sound waves) are reflected and refracted at the boundary of the magnetosphere in such a way that a net transfer of transverse momentum occurs, thus providing an effective viscous interaction between the turbulent exterior medium and the magnetosphere. In section 2 of this paper an examination is made of the rate of dissipation of energy and of the rate of circulation of material through the magnetosphere during a typical magnetic storm. It is shown in section 3, by means of a few simple arguments from ordinary viscous boundary layer theory, that these are consistent with the hypothesis of viscous interaction between the solar wind and the magnetosphere, and that the necessary viscous drag could be provided by the wave refraction mechanism.

It should be noted that in his most recent papers Piddington[20,21,22,23,24,25] has adopted essentially the point of view expounded in I, although there are some differences in the emphasis placed on the various points involved. In particular he invokes "a surface frictional force between the solar ion stream and the geomagnetic cavity"; thus the arguments developed in this paper are also relevant to Piddington's work.

Although the arguments presented here suggest that a wave refraction mechanism satisfies the various dynamical requirements of an effective viscosity, it should not be considered that the mechanism is necessarily exclusive of others or even dominant. There are, in fact, a number of processes which cause drag froces to be exerted on the surface layers of the magnetosphere. As pointed out above, the existence of surface instabilities cannot be ruled out on the basis of the data available at present; thus transport of surface tubes of force by the solar wind could result from mixing in the case of severe instability or from the production of surface ripples if the instability is not very pronounced. Another interesting possibility is that lines of force on the upstream face of the magnetosphere should actually break so that the free ends become fused into lines of force of the interplanetary magnetic field. Such broken lines would then be carried along by the solar wind and be laid over surface of the geomagnetic tail where they could eventually merge, thus becoming detached once again from the interplanetary magnetic field. This process is very similar to that envisaged by Dungey[26], and it appears therefore that Dungey's ideas, as well as those of Piddington, may complement rather than run counter to the views expressed in I.

2. The Electric Field and Energy Dissipation In the Magnetosphere During A Magnetic Storm

The total flow rate through the interior of the magnetosphere during a magnetic storm can be found quite easily if the total potential variation associated with the electric field producing the circulation is known. That is, it is necessary to estimate the potential difference existing between the points A and B in figures (1) and (2). This can be done in several ways, each of which suggests that the potential difference is probably of the order of 2×10^4 volts.

Radar observations of the motions of auroral ionization disclose that the east-west velocities may on occasion be as much as several km sec^{-1} [27,28,29]. A typical storm-time value would be $V \doteq 1$ km sec^{-1}, so that the associated electric field (E) is

$$E = VB \approx 5 \times 10^{-4} \text{ volts cm}^{-1}, \tag{1}$$

where the earth's magnetic field (B) is taken to be 0.5 gauss. Assuming that the width of the region in which such velocities occur is 200 km (equivalent to 2° of latitude), then the potential drop (ϕ_{AB}) is about 10^4 volts for each of the two loops of figure(2), thus

$$\phi_{AB} \approx 2 \times 10^4 \text{ volts}. \tag{2}$$

Hines[29] has pointed out that ϕ_{AB} must be of this magnitude if the circulation is to be able to penetrate (in spite of the rotational motion of the magnetosphere) to a geocentric equatorial distance of 4-5 earth radii (geomagnetic latitudes

60^{o} - 65^{o}) as it must at times. The fact that the bulk of the auroral luminosity occurs at altitudes of about 100 km suggests that the auroral primaries have energies of the order of 10-20 kev[30], as also do direct measurements [31,32,33] and observations of doppler shifts of Lyman α radiation[36,35]. Since $|e\phi_{AB}|$ is the maximum energy which can be given to a particle carrying charge e by the circulation alone the typical energies of auroral primaries can be considered to be consistent with the above value of ϕ_{AB}.

The points A and B in figure (2) are at a geomagnetic latitude of about 70^{o}, and are separated by about 4000 km. Thus the average speed across the polar cap is roughly $\phi_{AB}/$ (B x 4000 km) $\approx$ 250 meters sec^{-1}, and the time required for material to move from P to Q (in both figures (1) and (2)), a distance of about 4000 km, is therefore about 4 hours. This is roughly the time required for material freshly captured from the solar wind to be transported from the surface of the magnetosphere into the interior, and should be an indication of the duration of the initial phase of magnetic storms. However, if the magnetosphere is already well filled with particles with energies of a few kev, (as it might be in an extended period of relatively high geomagnetic activity), it should not be necessary to wait till fresh material arrives from outside before the main phase of a storm starts. Thus one might expect a rapid shortening of the initial phases of a series of closely spaced storms, and for short initial phases in general to be correlated with high magnetic K indices in the few days preceeding the storm sudden commencement.

The enhancement of the viscously-induced circulation in the magnetosphere (with its associated electric field) must take place almost immediately following the sudden commencement of a magnetic storm. Thus increased auroral and magnetic activity should occur throughout the initial phase, expecially if there is a relative abundance of particles with energies of a few kev already in the magnetosphere. Since the enhanced circulation is considered to lead to a build-up of the ring current one would expect the latter to be at maximum strength roughly at the time the former begins to decay, and that this behavior should be reflected in the variations of the corresponding magnetic indices.

An obvious way in which energy is dissipated during magnetic storms is by collisional excitation and ionization of atmospheric atoms and molecules as part of the auroral phenomenon. The rate of dissipation by auroral processes must equal the rate of input of energy by the primary particles, which has been shown by direct measurement to be typically 1-10 ergs cm^{-2} sec^{-1} and occasionally as much as 1000 ergs cm^{-2} sec^{-1}.[30,31,32] Assuming that auroral primaries are deposited at the rate of 1 erg cm^{-2} sec^{-1} over an area measuring 10^{o} in latitude (1000km) and 5000 km in longitude in both hemispheres, it is deduced that the rate of dissipation due to auroral processes (Φ_A) during a magnetic storm is roughly

$$\Phi_A \approx 10^{17} \text{ ergs sec}^{-1}. \tag{3}$$

There is perhaps an uncertainty of a factor 10 in this calculation.

Another important cause of energy dissipation is heating of the atmosphere due to ohmic losses associated with the Ds currents. The ohmic losses are ($\underline{j}.\underline{E}$) per unit volume, where $\underline{j}$ is the current vector; clearly only the Pederson (direct) current contributes to this scalar product since the Hall current is perpendicular to the electric field $\underline{E}$. The total current (I) typically associated with each segment of the Ds current system is about 3×10^5 amps (this is consistent with the adopted values of V and ϕ_{AB} if the electron density is of the order of 3×10^5 cm^{-3}). Since the integrated Pederson conductivity of the ionosphere is roughly 1/10 the Hall conductivity[36], the Pederson current associated with the Ds system is approximately I/10 for each segment. Most of the dissipation takes place in the auroral zone (over a longitudinal distance S, say) where the currents are very intense. Thus the total dissipation due to ohmic losses (Φ_{DS}) is of the order

$$\Phi_{DS} = (\underline{j}.\underline{E}) \times (\text{volume}) \approx 2SIVB/10 \approx 10^{17} \text{ ergs sec}^{-1} \tag{4}$$

where a factor 2 has been included to cover both hemispheres, and S has been taken to be 10^4 km. Once again the uncertainty in the calculation may be a factor 10.

Finally, dissipation of energy takes place with the decay of the ring current associated with the main phase of magnetic storms. The ring current is probably an indication of stressing of the geomagnetic field due to trapping of large numbers of relatively low energy ($\sim$20 kev) protons, and the dissipation may result from charge exchange between these protons and exospheric hydrogen atoms[37]. According to I the protons are captured from the solar wind and carried into the magnetosphere (with a consequent gain in energy) by the circulation sketched in figure (1). The total energy of the ring current protons can be calculated from the distortion of the geomagnetic field [38,39]; this may build up in a moderate magnetic storm to something like 10^{23} ergs over a period of about 12 hours, and the energy input rate (Φ_R) is therefore of the order

$$\Phi_R \approx 2 \times 10^{18} \text{ ergs sec}^{-1}. \tag{5}$$

The uncertainty in Φ_R is probably not as great as in Φ_A and Φ_{DS}, amounting perhaps to a factor 3.

Energy must be supplied at least at a rate ($\Phi_R + \Phi_{DS} + \Phi_R$) during a magnetic storm and from the above analysis it appears that a reasonable upper limit to the energy requirement would be $\Phi_M \approx 10^{19}$ ergs sec^{-1}, allowing for the roughness of the various calculations. Any theory of geomagnetic storms must be able to account for such an energy input rate, and it will be shown in section 3 that the viscous interaction hypothesis can do so comfortably.

As a preliminary to a theory invoking the solar wind as the cause of geomagnetic storms, it is necessary to show that the solar material which interacts with the earth's magnetosphere transports energy at a rate which is greater than Φ_M. During a magnetic storm the flux of solar wind protons can be expected to be of the order of 3×10^9 cm^{-2} sec^{-1} (42), and the speed of the wind to be about 1000 km sec^{-1}. Thus the total flux of energy passing through a circle of radius 15 R_e, which is roughly the cross section of the magnetosphere, is approximately 10^{22} ergs sec^{-1} and this exceeds Φ_M by a factor 10^3. (It should be understood that the velocity obtained from the time interval between the occurrence of a storm-producing flare and the associated sudden commencement, is the average velocity of the shock wave as it moves between the sun and the earth. The actual particle velocity immediately behind the shock wave is at most 3/4 of this, so that the energies may be less than half the values obtained using the disturbance velocity. Furthermore, the energies of particles which have passed through the standing shock wave shown in figure (1) are unlikely to be exactly the same as those in the undisturbed solar wind. Thus there is not necessarily any significant discrepancy between the observed energies of protons and the time delay of the magnetic storm which occurred during the flight of Explorer X[40]. However, a considerable increase in the particle energies may become apparent a few hours after the storm sudden commencement if the storm blast wave is driven sufficiently hard by the coronal explosion[41].)

3. The Viscous Interaction Hypothesis

In a steady state the rate at which magnetic flux passes through the interior of the magnetosphere (that is, between A and B in figure (1)), must equal the rate at which magnetic flux is dragged around the surface of the magnetosphere by the solar wind. This is equivalent to the statement that the magnitude of the potential difference between the points A and B and the interplanetary gas is $\phi_{AB}/2$. Assuming that the speed (U) of the magnetospheric material in the boundary layer is the same as that of the solar wind just outside, then the layer thickness (δ) can be found from

$$2UB'\delta = \phi_{AB} \approx 2 \times 10^4 \text{ volts}, \tag{6}$$

where B' is the magnetic field strength in the layer. U will be rather less than the solar wind speed, say 500 km sec^{-1}, while B' is approximately 50γ; thus

$$\delta \approx 400 \text{ km}. \tag{7}$$

If it is assumed that δ is the displacement thickness of a viscous boundary layer[43,44], then

$$\delta = O(\nu X/U)^{1/2}, \tag{8}$$

where ν is the kinematic viscosity and X the distance from the forward stagnation point. With the above values of U and δ, and putting $X = 10^5$ km, then the kinematic viscosity required is found to be

$$\nu \approx 10^{13} \text{ cm}^2 \text{ sec}^{-1} \tag{9}$$

which is comparable with a value obtained under a different set of assumptions by Parker[11]. The Reynolds number appropriate to the situation is

$$R = Ua/\nu , \tag{10}$$

where a is a typical dimension of the magnetosphere; taking $a = 10^5$ km, then $R \approx 5 \times 10^4$, which is large compared to unity and hence the use of boundary layer theory is a reasonable procedure.

The rate at which energy is transferred to the magnetosphere by such a viscous interaction is approximately $\rho U^2 (\nu U/X)^{1/2}$ per unit area, where ρ is the density of the material in the outer fringe of the magnetosphere. Thus a rough value for the total energy input (Φ_V) is given by

$$\Phi_V \approx \rho U^2 A(\nu U/X)^{1/2}, \tag{11}$$

where A is taken to be the area of a sphere of radius 15 R_e, and $\rho = 2 \times 10^{-23}$ gm cm^{-3}, corresponding to about 10 protons cm^{-3}. With the above values for U, ν and X, it is found that

$$\Phi_V \approx 10^{19} \text{ ergs sec}^{-1}. \tag{12}$$

Despite the uncertainties necessarily entailed in these calculations, it is remarkable that the value of Φ_V, which has been calculated using quite acceptable values of ϕ_{AB}, U, X, B' and ρ, is consistent with the dissipation rate Φ_M. It is emphasized that none of the quantities used to calculate Φ_V has been used in determining Φ_M, which is essentially an observed quantity. Thus it may be considered that the viscous interaction hypothesis implies that dissipation at a rate of the order of Φ_M must occur if $\phi_{AB} \approx 20$ kilovolts.

It is now necessary to show that viscous stresses of the required magnitude can be realized. The usual mechanisms for the transfer of transverse momentum in a shear flow involve molecular diffusion or turbulent mixing, and these may not be very effective in the circumstances under consideration. However, as remarked earlier, momentum transferred by sound waves provides a possible source of drag, as the solar wind near the surface of the magnetosphere is known to be in a state of compressible turbulence. Consider the situation depicted in figure (3): the fluid in region (1) is stationary and contains an isotropic assemblage of sound waves with an energy density ξ, while the fluid in region (2) moves with velocity in the x direction. If = 0, a pair of equal waves which approach the interface (y=0) at angles $\pm\ \theta$ to the y axis are reflected and refracted in a symmetrical manner, therefore the x components of the radiation pressures cancel. This is true for all values of θ, and hence for an isotropic flux of waves from region (1). Although there is no transverse stress in this situation the sound waves do carry momentum and energy normal to the interface into region (1). However, if $\neq 0$, then equal waves with angles of incidence $\pm\ \theta$ are not reflected and refracted in the same manner, hence the components of the radiation pressures of the two refracted waves do not exactly cancel and there is a residual stress in the x direction which is in fact a drag.

The problem of reflection and refraction of hydromagnetic waves at a shear layer has been examined by Fejer[45], allowing for different densities and sound velocities on each side of the interface; the problem is rather complicated and it is doubtful whether the results can be usefully applied to the interface between the solar wind and the magnetosphere. However, it seems sufficient for the purposes of this calculation to treat both media as ordinary compressible fluids with equal densities and sound velocities. This case has been examined by Ribner[46] who shows that the asymmetry described above is very pronounced expecially when the Mach number M = /c (where c is the speed of sound) is comparable to or greater than unity. When $M \leq 2$, the asymmetry is due largely to the total reflection of waves for which θ is greater than a certain positive value which decreases with increasing M. When $M > 2$ a remarkable self-excited oscillation of the interface takes place, such that the transmission and reflection coefficients exceed unity and even become infinite for some values of θ. Taking $M = 2$, a value which is consistent with the findings of Explorer X[47], it is found from Ribner's results that the rate of transfer of energy per unit area is approximately $0.2c\xi$, and the drag per unit area (D_W) is appromimately $D_W = 0.2\xi$.

For the situation under consideration, the velocity of sound must be approximately equal to the Alfvén speed in the outer fringes of the magnetosphere, thus $c \approx (B^2/4\pi\rho)^{1/2}$ ≈ 250 km $\sec^{-1}$, and $\approx U = 500$ km $\sec^{-1}$. Sonett and Abrams[18] have deduced that $\xi \approx 10^{-9}$ ergs cm^{-3} for non-storm conditions on the upstream side of the magnetosphere, and this will be adopted as a working value for stormy conditions around the whole magnetosphere. It is assumed that the energy transfer takes place over an area comparable to that of a sphere of radius equal to 15 R_e, thus the total rate of energy transfer by sound waves (Φ_W) can be expected to be of the order

$$\Phi_W \approx 5 \times 10^{18} \text{ ergs sec}^{-1}, \qquad (13)$$

while the drag is approximately

$$D_W \approx 2 \times 10^{-10} \text{ dynes cm}^{-2}. \tag{14}$$

One cannot derive an equivalent kinematic viscosity from D_W however the drag per unit area corresponding to the kinematic viscosity deduced above is

$$D_V \approx \nu\rho U/\delta \approx 2 \times 10^{-10} \text{ dynes cm}^{-2}. \tag{15}$$

The agreement between the calculated values of Φ_W and Φ_V, and between D_W and D_V, suggests that the sound wave mechanism can indeed form the basis of a sufficiently strong viscous interaction. There are, of course, many uncertainties involved in the calculations and the close agreement shown must be largely coincidental; however, there is no apparent a priori reason that the quantities should agree at all since the observed value of ξ is quite independent of the values of all other parameters. We therefore gain confidence in the suggestion that the electric fields within the magnetosphere associated with polar current systems, are the direct result of a viscous-like interaction with the solar wind and the surface of the magnetosphere. This has an advantage in that the immediate cause of all the phenomena is readily understood to be the solar wind, whereas the mechanisms which involve charge separation due to differential drifts must assume the existence of an undefined additional process to provide the trapped radiation in the first place.

It is rather difficult to suggest experiments which would show conclusively that the above arguments are correct, however there is some indirect evidence that viscous interaction between the solar wind and the surface of the magnetosphere does take place. The interplanetary magnetic field tends to produce a uniform pressure outside the magnetosphere which according to some authors should tend to close the magnetosphere at about 20 R_e on the downstream side[48,49]. Observations from Explorer X indicate that the magnetosphere shows no sign of closing at 40 R_e[15,47]. This might be interpreted as an indication that viscous stresses exert an important influence on the shape of the magnetosphere, particularly on the downstream side where lines of force may be dragged out to 60 R_e or more[16,26]. The large diurnal variation in the latitude of termination of trapping of electrons with energies greater than 40 kev[50], could be at least partly due to lengthening of the geomagnetic field lines on the night side of the earth as a result of viscous drag on the tail of the magnetosphere, although the circulation sketched in figure (1) could produce a similar effect by driving the electrons towards lower latitudes in this region.

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Figure (1)

A sketch of the equatorial section of the earth's magnetosphere looking from above the north pole. Streamlines of the solar wind are shown on the exterior, and the internal streamlines refer to the circulation which it is proposed is set up by viscous interaction between the solar wind and the surface of the magnetosphere. The internal streamlines are also equipotentials of an associated electric field which may be regarded as being due to accumulations of positive and negative charges as indicated at A and B.

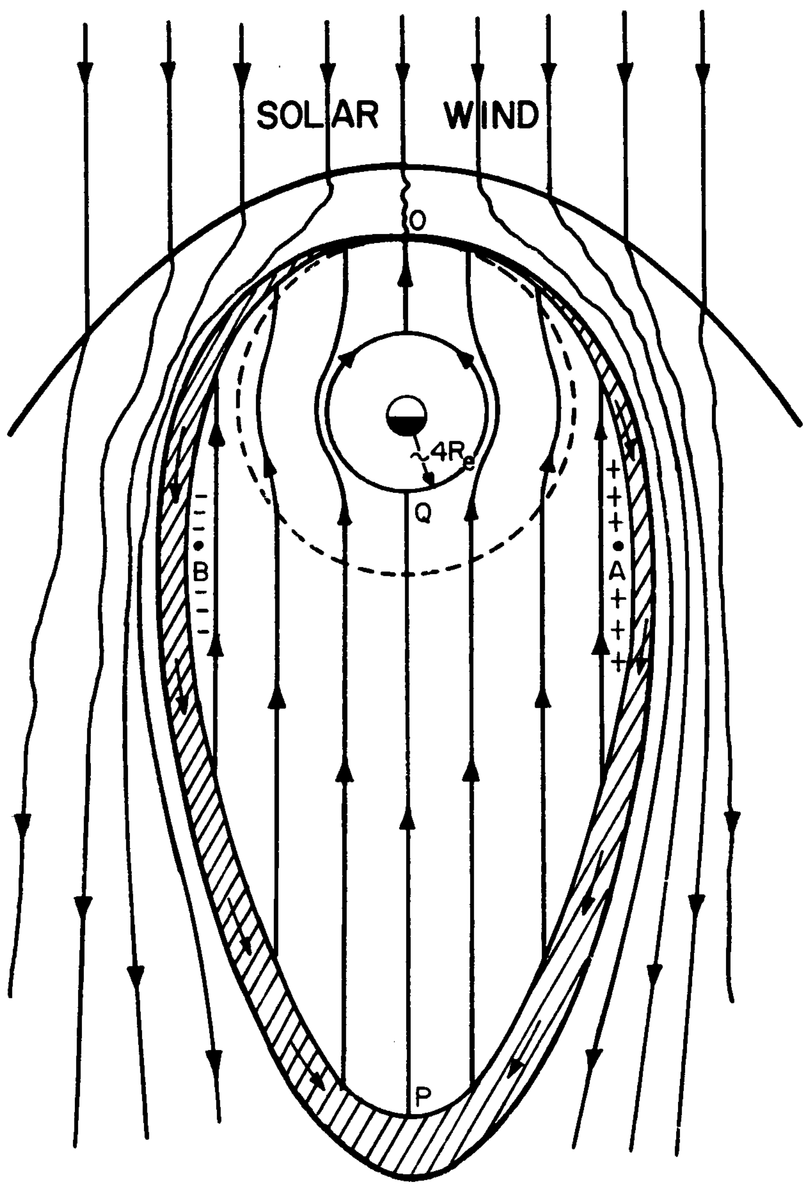

SOLAR WIND
O
~4R_e
Q
B
A
P

Figure (2)

A sketch of the circulation at ionospheric levels in the north polar cap corresponding to the internal circulation in the magnetosphere shown in figure (1).

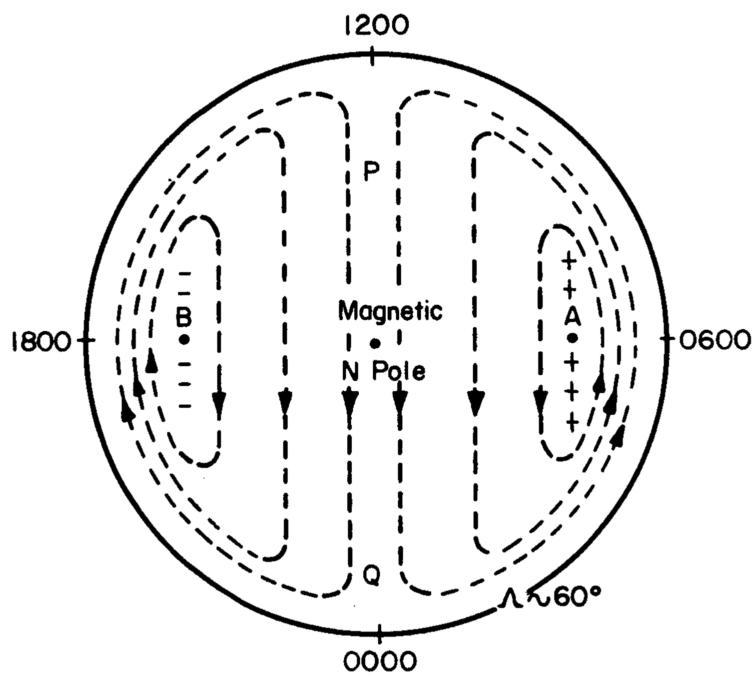
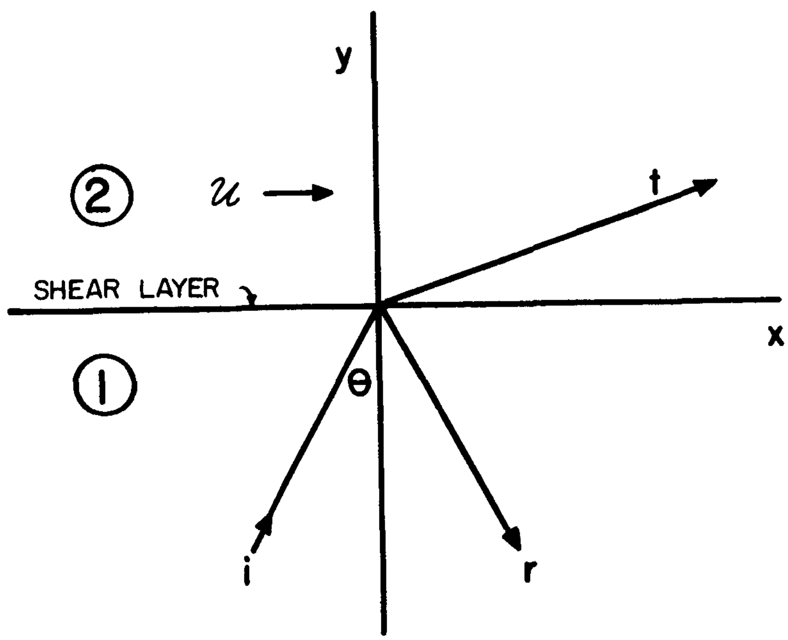


Figure (3)

Refraction and reflection of a sound wave incident on a velocity discontinuity. The fluid in $y<0$ (region (1)) is assumed to be stationary while that in $y>0$ (region (2)) moves in the x direction with velocity . A sound wave (i) is incident on the shear layer $y = 0$ from region (1) giving rise to a reflected wave (r) and a refracted or transmitted wave (t).



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